# MA114 Summer II 2018
# Worksheet 3a – Trig Integrals
# 6/12/18

Solution

1. Compute the following integrals.

a) $\int_0^{\pi/2} \cos^3(x)\ dx$, $= \int_0^{\pi/2} \cos^2(x)\cos(x)\,dx$

$= \int_0^{\pi/2} (1-\sin^2(x))\cos(x)\,dx$ $\quad u=\sin(x)\quad du=\cos(x)\,dx$

$x=0: u=0 \qquad x=\pi/2 \quad u=1$

$= \int_0^1 1-u^2\,du$

$= u-\frac{1}{3}u^3\Big|_0^1 = \boxed{\frac{2}{3}}$

b) $\int \frac{\sin(x)}{\cos^3(x)}\ dx$, $\quad u=\cos(x)\quad du=-\sin(x)\,dx$

$= \int -\frac{1}{u^3}\,du = \int -u^{-3}\,du = \frac{1}{2}u^{-2}+C = \boxed{\frac{1}{2\cos^2(x)}+C}$

c) $\int \sqrt{\cos x}\sin^3(x)\ dx$, $= \int \sqrt{\cos(x)}\ \sin^2(x)\sin(x)\,dx$

$= \int \sqrt{\cos(x)}\ (1-\cos^2(x))\sin(x)\,dx$ $\quad u=\cos(x)\quad du=-\sin(x)\,dx$

$= \int -\sqrt{u}\,(1-u^2)\,du$

$= \int u^{5/2}-u^{1/2}\,du$

$= \frac{2}{7}u^{7/2}-\frac{2}{3}u^{3/2}+C$

$= \boxed{\frac{2}{7}\cos^{7/2}(x)-\frac{2}{3}\cos^{3/2}(x)+C}$

d) $\int_0^{2\pi} \sin^2(\frac{1}{3}\theta)\ d\theta$. $= \int_0^{2\pi} \frac{1}{2}\left(1-\cos\left(\frac{2}{3}\theta\right)\right)d\theta$

$= \frac{1}{2}\left[\theta-\frac{3}{2}\sin\left(\frac{2}{3}\theta\right)\right]\Big|_0^{2\pi}$

$= \frac{1}{2}\left[2\pi-\frac{3}{2}\sin\left(\frac{4\pi}{3}\right)-0+\frac{3}{2}\sin(0)\right]$

$= \frac{1}{2}\left[2\pi+\frac{3\sqrt{3}}{4}\right]$

$= \boxed{\pi+\frac{3\sqrt{3}}{8}}$

$I$
$\parallel$

2. Evaluate $\int \sin x \cos x \, dx$ by four methods:

   a) the substitution $u = \cos x$

   b) the substitution $u = \sin x$

   c) the identity $\sin 2x = 2 \sin x \cos x$

   d) integration by parts.

Explain the different appearances of the answers. How are they related?

a) $I = \int -u \, du$
$= -\frac{1}{2}u^2 + C$
$= \underline{-\frac{1}{2}\cos^2(x) + C_1}$

c) $I = \frac{1}{2}\int \sin(2x)\, dx$
$= \underline{-\frac{1}{4}\cos(2x) + C_3}$

b) $I = \int u \, du$
$= \underline{\frac{1}{2}\sin^2(x) + C_2}$

d) Same as a) or b) depending on choice of $u$, $dv$.

See $\boxed{\text{desmos.com/calculator/tbdynchots.}}$

3. Consider integrating $\sin^m(x)\cos^n(x)$ with respect to $x$. When are each of the following strategies useful? (Think about whether $m$ and $n$ are even or odd.)

   a) Save a power of $\cos(x)$, use the Pythagorean identity to convert the others into powers of $\sin(x)$, then substitute $u = \sin(x)$.

   b) Use the half-angle formulas $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$ and $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$.

   c) Save a power of $\sin(x)$, use the Pythagorean identity to convert the others into powers of $\cos(x)$, then substitute $u = \cos(x)$.

a) Useful when <u>n is odd</u> (so we can save 1 power of cos and group the rest into pairs of two.

c) Useful when <u>m is odd</u> (same as a) but for sin)

b) Useful when <u>both n and m are even</u>.